

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
International GCSE**

Centre Number

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Candidate Number

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Time 2 hours

Paper
reference

4PM1/01

**Further Pure Mathematics
PAPER 1**



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ►

P66024A

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \frac{a}{1 - r}$ $|r| < 1$

Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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2 Angle α is acute such that $\cos \alpha = \frac{3}{5}$

Angle β is obtuse such that $\sin \beta = \frac{1}{2}$

(a) Find the exact value of

(i) $\tan \alpha$

(ii) $\tan \beta$

(3)

(b) Hence show that

$$\tan(\alpha + \beta) = \frac{m\sqrt{3} - n}{n\sqrt{3} + m}$$

where m and n are positive integers whose values are to be found.

(3)



Question 2 continued

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(Total for Question 2 is 6 marks)



3 A curve C has equation $y = \frac{ax - 3}{x + 5}$ where a is a constant and $x \neq -5$

The gradient of C at the point on the curve where $x = 2$ is $\frac{18}{49}$

(a) Show that $a = 3$ (3)

Hence

(b) write down an equation of the asymptote to C that is
(i) parallel to the x -axis,
(ii) parallel to the y -axis, (2)

(c) find the coordinates of the point where C crosses
(i) the x -axis,
(ii) the y -axis. (2)

(d) Sketch the curve C , showing clearly its asymptotes and the coordinates of the points where C crosses the coordinate axes. (3)

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Question 3 continued

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Question 3 continued

Handwriting practice area consisting of 25 horizontal dotted lines.

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Question 3 continued

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(Total for Question 3 is 10 marks)



4 The n th term of an arithmetic series is u_n where

$$u_n = (n + 1) \ln 4$$

Given that the sum of the first n terms of the series is S_n

show that $S_n = \ln 2^{(n^2 + an)}$ where a is an integer whose value is to be found.

(5)

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Question 4 continued

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(Total for Question 4 is 5 marks)



Question 5 continued

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(Total for Question 5 is 10 marks)



Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)



Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 11 marks)



8 Given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

(a) find the value of n .

(3)

Given that $\log_p x = 3$ and $\log_p y - 3 \log_p 2 = 4$

(b) (i) express x in terms of p ,

(1)

(ii) express xy in terms of p .

(4)



Question 8 continued

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Question 8 continued

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(Total for Question 8 is 8 marks)



9 Find an equation of the normal to the curve with equation

$$y = (x^3 - 2x)e^{(1-x)}$$

at the point on the curve with coordinates $(1, -1)$

(5)

Area with horizontal dotted lines for writing the solution.

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Question 9 continued

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(Total for Question 9 is 5 marks)



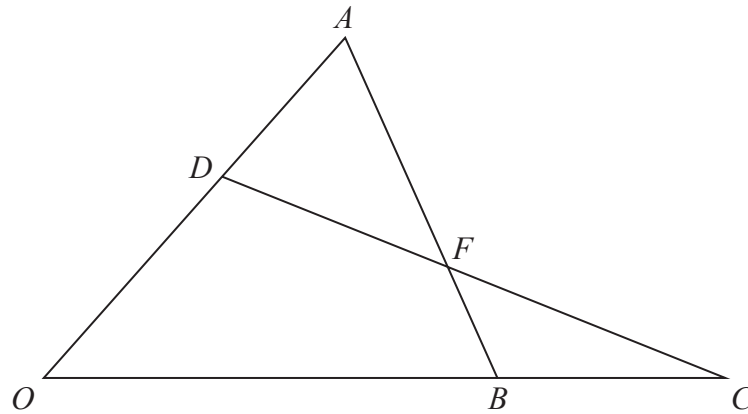


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows triangle OAB and triangle OCD .

$$\vec{OA} = 5\mathbf{p} \quad \vec{AB} = 3\mathbf{q} \quad \vec{OC} = \frac{3}{2}\vec{OB} \quad \vec{OD} = \frac{3}{5}\vec{OA}$$

(a) Find \vec{DC} as a simplified expression in terms of \mathbf{p} and \mathbf{q} .

(3)

The line DC meets the line AB at F .

(b) Using a vector method, find \vec{OF} as a simplified expression in terms of \mathbf{p} and \mathbf{q} .

(7)

The point G lies on OB such that FG is parallel to AO .

(c) Using a vector method, find \vec{OG} as a simplified expression in terms of \mathbf{p} and \mathbf{q} .

(4)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 14 marks)



11 (a) Using a formula from page 2, show that $\cos 2x = 1 - 2 \sin^2 x$

(3)

Diagram **NOT** accurately drawn

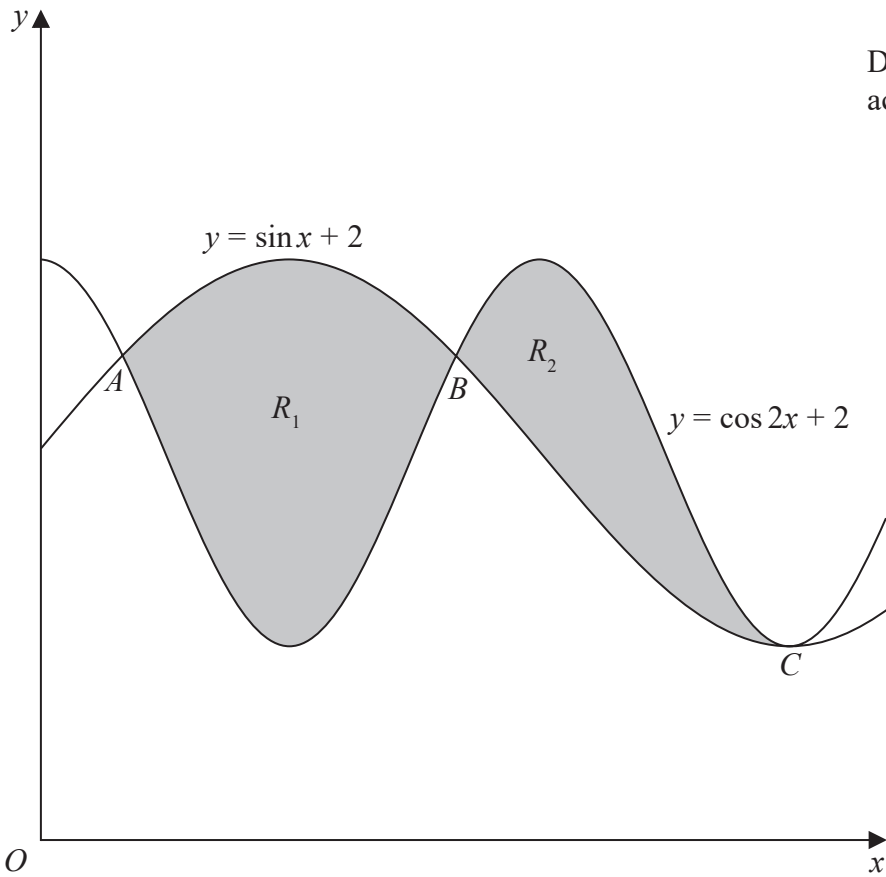


Figure 2

Figure 2 shows a sketch of part of the curves with equations $y = \sin x + 2$ and $y = \cos 2x + 2$

The points A , B and C , shown in Figure 2, are three points that are common to both curves.

(b) Find the coordinates of each of these points.

(4)

R_1 and R_2 , shown shaded in Figure 2, are two regions enclosed by the two curves.

(c) Use calculus to find, in its simplest form, the ratio

$$\text{area of } R_1 : \text{area of } R_2$$

(8)

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Question 11 continued

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Question 11 continued

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